

Stability of dark matter from the $D_4 \times Z_2^f$ flavor group

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We study a model based on the dihedral group D_4 in which the dark matter is stabilized by the interplay between a remnant Z_2 symmetry, of the same spontaneously broken non-abelian group, and an auxiliary Z_2^f introduced to eliminate unwanted couplings in the scalar potential. In the lepton sector the model is compatible with normal hierarchy only and predicts a vanishing reactor mixing angle, $\theta_{13} = 0$. Since $m_{\nu_1} = 0$, we also have a simple prediction for the effective mass in terms of the solar angle: $|m_{\beta\beta}| = |m_{\nu_2}| \sin^2 \theta_\odot \sim 10^{-3}$ eV. There also exists a large portion of the model parameter space where the upper bounds on lepton flavor violating processes are not violated. We incorporate quarks in the same scheme finding that a description of the CKM mixing matrix is possible and that semileptonic K and D decays mediated by flavor changing neutral currents are under control.

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I. INTRODUCTION

We have strong evidence about the existence of dark matter (DM) [1, 2]. A good DM candidate must be neutral and stable or with a decay length bigger than the age of the universe and give the correct relic abundance [3]. There are several extensions of the standard model predicting good DM candidates; however, it turns out that in many models the stability of the DM is obtained introducing ad-hoc assumptions, see for example the review [4]. Any of these models may be correct but certainly it would be desirable to provide a fundamental explanation of the origin of the stability. In [5] it has been pointed out that the stability can be guaranteed by a residual Z_2 symmetry arising from the spontaneous breaking of a non-abelian flavor symmetry; the same Z_2 also acts in the neutrino sector and has a strong impact on the phenomenology of neutrino masses and mixing. In that model the flavor symmetry is the group of the even permutations of four objects A_4 whose irreducible representations are three singlets and one triplet. To avoid a direct couplings to quarks and charged leptons, the DM candidate is assigned to a triplet representation, while leptons and quarks to singlets of A_4 . After electroweak symmetry breaking, A_4 is broken into its subgroup Z_2 under which two component of the triplet DM are automatically charged; eventually, this prevents dangerous couplings with the Higgs fields of the model. Such an idea has been then further studied and extended in refs. [6, 7].

The interplay between decaying dark matter and non-abelian discrete flavor symmetries has been considered in a number of subsequent papers; for instance, in [8–10] non-abelian discrete symmetries prohibit operators that may induce too fast dark matter decay; in [11] a non-abelian discrete symmetry (not a flavor symmetry) has been used to stabilize the scalar DM candidate (similar to what has been discussed in the inert scalar models [12]) and the matter sector has not been considered. Therefore the models in [8–10] and [11] are substantially different to the idea

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introduced in [5].

In this paper we adopt the point of view elucidated in [5–7], studying a flavor model where the stability of the DM is caused by the interplay between a remnant Z_2 symmetry of the D_4 group and an auxiliary Z_2^f which allows to eliminate dangerous couplings in the scalar potential. Since D_4 contains only singlets and doublets it is highly non-trivial to still be able to generate the mechanism for dark matter stabilization; in addition, the same non-abelian symmetry also acts on leptons and quarks, giving acceptable phenomenology in both sectors.

The relevant differences of our model compared to [5–7] can be summarized as follows:

- for the first time, we extend such a mechanism to incorporate quarks transforming under non-trivial representation of D_4 ; in this framework, we are able to reproduce the correct order of magnitude of the quark mixing angles, a quite remarkable result;
- charged leptons are non-diagonal (with hierarchy among the eigenvalues naturally reproduced with $\mathcal{O}(1)$ Yukawa couplings) and completely responsible for the atmospheric mixing angle in the neutrino sector, instead of being diagonal as in [5–7];
- although the Higgs sector is extended with three more scalar doublets and one singlet, the neutrino sector contains only two right-handed neutrinos. The model can be considered minimal in this respect.

The paper is organized as follows: in section II we present the relevant features of the model, discussing the group properties of D_4 and the assignments of leptons and Higgs fields to the irreducible representations of the group. In section III we discuss the scalar potential of the theory and describe in details how the DM stability arises in our model; sections IV and V are devoted to the neutrino phenomenology and to the estimate of some relevant lepton flavor violating processes, respectively. In section VI we discuss the quark sector and give an order of magnitude estimate of some of the flavor changing neutral current processes; eventually, in section VII we draw our conclusions.

II. MODEL

We assign the fields of the model into irreducible representation of D_4 , the dihedral group of order four [13], see also [14]. It has five irreducible representations, four singlets $1_{1,2,3,4}$ and one doublet 2 . The generators of the group fulfill the relations:

$$A^4 = B^2 = 1, \quad ABA = B. \quad (1)$$

The one-dimensional representations are characterized by $A = B = 1$ for 1_1 , $A = 1$, $B = -1$ for 1_2 , $A = -1$, $B = 1$ for 1_3 and $A = -1$, $B = -1$ for 1_4 . The generators for the two-dimensional representations are

$$A = \begin{pmatrix} i & 0 \\ 0 & -i \end{pmatrix}, \quad B = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}. \quad (2)$$

An interesting feature of D_4 is that the product of two doublets contains only singlets: given $(a_1, a_2) \sim 2$ and $(b_1, b_2) \sim 2$ we have:

$$\begin{aligned} a_1 b_2 + a_2 b_1 &\sim 1_1, & a_1 b_2 - a_2 b_1 &\sim 1_2, \\ a_1 b_1 + a_2 b_2 &\sim 1_3, & a_1 b_1 - a_2 b_2 &\sim 1_4. \end{aligned} \quad (3)$$

For the singlets: $1_i \times 1_i = 1_1$, $1_1 \times 1_i = 1_i$ for $i = 1, \dots, 4$, $1_2 \times 1_3 = 1_4$, $1_2 \times 1_4 = 1_3$ and $1_3 \times 1_4 = 1_2$. The standard model Higgs doublet is taken as a singlet 1_1 ; we assume three further Higgs doublets, one of them transforming as a singlet 1_3 (H') and the other as a doublet of D_4 , $\eta = (\eta_1, \eta_2) \sim 2$. In order to correctly describe both lepton and quark sectors, we need to introduce a scalar $SU(2)$ singlet flavon ϕ in the 1_2 representation. Two right-handed neutrinos (N_1, N_2) in the doublet representation N_D are necessary ingredients to give mass to the neutrinos via the

	L_e	L_μ	L_τ	l_e^c	l_μ^c	l_τ^c	N_D	H	H'	η	ϕ
$SU(2)$	2	2	2	1	1	1	1	2	2	2	1
D_4	1 ₁	1 ₂	1 ₃	1 ₄	1 ₁	1 ₃	2	1 ₁	1 ₃	2	1 ₂
Z_2^f	+	+	+	+	-	+	+	+	-	+	-

TABLE I: Assignment of the lepton and Higgs fields under $SU(2)$, D_4 and Z_2^f .

type-I seesaw mechanism. The assignment to the irreducible representations of D_4 as well as the charges under Z_2^f are listed in Tab.I.

The invariant $D_4 \times Z_2^f$ Lagrangian in the lepton sector is as follow:

$$\begin{aligned} \mathcal{L} = & \frac{y_1^l}{\Lambda} L_e l_e^c H' \phi + \frac{y_2^l}{\Lambda} L_\mu l_\mu^c H \phi + y_3^l L_\tau l_\tau^c H + \frac{y_4^l}{\Lambda} L_\mu l_\tau^c H' \phi + y_5^l L_\tau l_\mu^c H' + \\ & y_1^\nu L_e (N_D \eta)_{11} + y_2^\nu L_\mu (N_D \eta)_{12} + y_3^\nu L_\tau (N_D \eta)_{13} + \\ & + M_1 N_D N_D + \text{h.c.}, \end{aligned} \quad (4)$$

where Λ is a large energy scale. Here we have only considered terms up to one-flavon insertion. It turns out that higher powers of Higgs fields and flavon insertions can only modify (to a negligible level) the couplings y_i^l but are not able to generate new Yukawa interactions. In the neutrino sector, due to the Z_2^f symmetry, the previous lagrangian is modified by operators with at least two powers of ϕ , that we assume here negligible. We will investigate in details its phenomenological consequences in sections IV and V.

III. SCALAR SPECTRUM AND STABILITY OF THE DARK MATTER CANDIDATE

The invariant scalar potential is of the form:

$$\begin{aligned} V = & \mu_H^2 H^\dagger H + \mu_{H'}^2 H'^\dagger H' + \mu_\eta^2 (\eta^\dagger \eta)_{11} + \mu_\phi \phi^2 + \\ & + \lambda_1 (\eta^\dagger \eta)_{11} (\eta^\dagger \eta)_{11} + \lambda_2 (\eta^\dagger \eta)_{12} (\eta^\dagger \eta)_{12} + \lambda_3 (\eta^\dagger \eta)_{13} (\eta^\dagger \eta)_{13} + \lambda_4 (\eta^\dagger \eta)_{14} (\eta^\dagger \eta)_{14} + \\ & + \lambda'_1 (\eta^\dagger \eta^\dagger)_{11} (\eta \eta)_{11} + \lambda'_2 (\eta^\dagger \eta^\dagger)_{12} (\eta \eta)_{12} + \lambda'_3 (\eta^\dagger \eta^\dagger)_{13} (\eta \eta)_{13} + \lambda'_4 (\eta^\dagger \eta^\dagger)_{14} (\eta \eta)_{14} + \lambda_5 (\eta^\dagger \eta)_{11} H^\dagger H + \\ & + \lambda'_5 (\eta^\dagger H) (H^\dagger \eta) + \lambda_6 [(\eta^\dagger \eta^\dagger)_{11} H H + \text{h.c.}] + \lambda_7 (\eta^\dagger \eta)_{11} H'^\dagger H' + \lambda'_7 (\eta^\dagger H') (H'^\dagger \eta) + \\ & + \lambda_8 [(\eta^\dagger \eta^\dagger)_{11} H' H' + \text{h.c.}] + \lambda_9 (H'^\dagger H') (H^\dagger H) + \lambda'_9 (H'^\dagger H) (H^\dagger H') + \\ & + \lambda_{10} [(H'^\dagger H) (H'^\dagger H) + \text{h.c.}] + \lambda_{11} (H^\dagger H) (H^\dagger H) + \lambda_{12} (H'^\dagger H') (H'^\dagger H') + \\ & + \xi_1 \phi^2 H^\dagger H + \xi_2 \phi^2 H'^\dagger H' + \xi_3 \phi^2 (\eta^\dagger \eta)_{11} + \xi_4 \phi^4. \end{aligned} \quad (5)$$

We assume a vev structure of the form:

$$\langle H \rangle = v, \quad \langle H' \rangle = v', \quad \langle \eta_1 \rangle = v_{\eta_1}, \quad \langle \eta_2 \rangle = v_{\eta_2}, \quad \langle \phi \rangle = v_\phi \quad (6)$$

where the various vevs v_i are obtained solving the coupled differential equations $\partial V / \partial v_i = 0$. Assuming for simplicity real vevs, we have carefully checked that, for suitable parameter choices of the potential V , an allowed local minimum is:

$$v_{\eta_1} = v_{\eta_2} = v_\eta, \quad (7)$$

which is the crucial point to justify the stability of the DM based on symmetry arguments. After electroweak symmetry breaking we can write:

$$\eta_1 = \begin{pmatrix} \eta_1^+ \\ v_\eta + \eta'_1 + iA_1 \end{pmatrix}, \quad \eta_2 = \begin{pmatrix} \eta_2^+ \\ v_\eta + \eta'_2 + iA_2 \end{pmatrix}, \quad (8)$$

$$H = \begin{pmatrix} H^+ \\ v + H + iA \end{pmatrix}, \quad H' = \begin{pmatrix} H'^+ \\ v' + H' + iA' \end{pmatrix}, \quad (9)$$

and the physical spectrum involves four neutral scalars, three pseudoscalars and three charged scalars (plus one flavon). To maintain the notation compact and avoid unnecessary complications, we work in the limit of decoupled ϕ (that is $\xi_{1,2,3} = 0$), which does not modify any of the results discussed in the paper (and, of course, the vev alignment $\langle\eta\rangle \sim (1, 1)$). In such a limit, the mass matrices of the three sectors (S =scalar, A =pseudoscalar, H^+ =charged) can be generically written in the following way:

$$(M^{S,A,H^+})^2 = \begin{pmatrix} M_{11}^{S,A,H^+} & M_{12}^{S,A,H^+} & M_{13}^{S,A,H^+} & M_{13}^{S,A,H^+} \\ M_{12}^{S,A,H^+} & M_{22}^{S,A,H^+} & M_{23}^{S,A,H^+} & M_{23}^{S,A,H^+} \\ M_{13}^{S,A,H^+} & M_{23}^{S,A,H^+} & M_{33}^{S,A,H^+} & M_{34}^{S,A,H^+} \\ M_{13}^{S,A,H^+} & M_{23}^{S,A,H^+} & M_{34}^{S,A,H^+} & M_{33}^{S,A,H^+} \end{pmatrix}. \quad (10)$$

The relevant feature here is that the 2×2 sub-block corresponding to the $3 - 4$ sector is symmetric and can be put in a block diagonal form by a maximal rotation. This corresponds to a rotation in the corresponding bidimensional subspace which defines the mass eigenstates of the subsector. After this change of basis, we are left with block-diagonal mass matrices made by 3×3 matrices (one for scalars, one for pseudoscalars and one for charged), and 1×1 blocks corresponding to the isolated DM sector:

$$M^{S,A,H^+} = \begin{pmatrix} M_{11}^{S,A,H^+} & M_{12}^{S,A,H^+} & \sqrt{2}M_{13}^{S,A,H^+} & 0 \\ M_{12}^{S,A,H^+} & M_{22}^{S,A,H^+} & \sqrt{2}M_{23}^{S,A,H^+} & 0 \\ \sqrt{2}M_{13}^{S,A,H^+} & \sqrt{2}M_{23}^{S,A,H^+} & M_{33}^{S,A,H^+} + M_{34}^{S,A,H^+} & 0 \\ 0 & 0 & 0 & (m_{DM}^{S,A,H^+})^2 \end{pmatrix}, \quad (11)$$

where

$$(m_{DM}^{S,A,H^+})^2 = M_{33}^{S,A,H^+} - M_{34}^{S,A,H^+}. \quad (12)$$

The explicit expressions of the physical masses are complicated functions of the potential parameters and their expressions do not reveal any important features to be mentioned here beside the fact that, as expected, the pseudoscalars and charged mass matrices have a zero eigenvalues corresponding to the Goldstone bosons.

We have verified that, for a suitable set of the potential parameters, m_{DM}^S is the lightest mass of the neutral stable states and it is smaller than the corresponding charged states carrying the same Z_2 parity; in addition, all the masses arising from the diagonalization of the 3×3 matrices in eq.(11) are positive and do not violate the electroweak constraints on the T , S and U oblique parameters[15].

We are now in the position to discuss the mechanism for the DM stability in our model. The condition in eq.(6) can be rewritten as:

$$\langle H \rangle = v, \quad \langle H' \rangle = v', \quad \langle \eta \rangle = v_\eta \begin{pmatrix} 1 \\ 1 \end{pmatrix}. \quad (13)$$

The generator B of D_4 acts as the identity on the singlet representations (1_1 for H and 1_3 for H'). On the other hand, from equation (2) we also see that B , which is the generator of a Z_2 symmetry, leaves invariant the vector $\langle\eta\rangle$.

In a compact 4×4 form, such a generator can be written as:

$$B_{4 \times 4} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}, \quad (14)$$

where it is understood that the first two entries work on the H and H' fields and the others on the components of the η field. After performing the $3 - 4$ rotation that diagonalizes the corresponding entries in the mass matrices, the generator can be cast in a diagonal form:

$$B_{4 \times 4} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}, \quad (15)$$

showing the peculiar feature of the negative value in the $(4, 4)$ entry. The previous 2×2 rotation defines the mass eigenstates in the D_4 doublet subspace of eq.(8):

$$\begin{aligned} \eta_p &= \frac{1}{\sqrt{2}}(\eta_2 + \eta_1), \\ \eta_m &= \frac{1}{\sqrt{2}}(\eta_2 - \eta_1), \end{aligned} \quad (16)$$

that, under the remnant Z_2 symmetry defined in eq.(15), transform as

$$\eta_p \rightarrow +\eta_p, \quad \eta_m \rightarrow -\eta_m, \quad (17)$$

with vevs¹

$$\langle \eta_m \rangle = 0, \quad \langle \eta_p \rangle = \sqrt{2}v_\eta. \quad (18)$$

The same conclusions can be drawn working in a different basis where

$$A = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}, \quad B = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}. \quad (19)$$

In that case, η develops a vev along the direction $(1, 0)$ and the component that does not take vev is charged under the remaining Z_2 parity².

Let us now make more transparent the role of the Z_2^f . If we introduce linear and/or trilinear terms in the scalar potential transforming as 1_2 and 1_4 representations (like ϕ in our case), the vev of η would not be aligned along the direction $(1, 1)$ and the residual Z_2 would be broken, causing the decay of the DM. In fact, one can add contractions of the form $(\eta^\dagger \eta)_{1_2}$ into 1_2 singlet of D_4 and the minimizing equations for the η components would admit solutions only if $\langle \eta_1 \rangle \neq \langle \eta_2 \rangle$; the auxiliarly symmetry Z_2^f (under which $\phi \rightarrow -\phi$) only allows quadratic and quartic terms in ϕ which transform as a singlet 1_1 and avoid the dangerous Z_2 -breaking contractions.

We see that η_m is the only Higgs field charged under the Z_2 symmetry. This prevents any coupling with other Higgs fields and, considering also that the original η field does not couple to quarks and charged lepton bilinears (but only to heavy right-handed neutrinos), the component of η_m corresponding to the lightest Z_2 -odd neutral spin zero particle is the DM candidate of the model, namely the combination $\eta'_1 - \eta'_2$.

Since the DM couplings with the standard model particles are very similar to the ones in ref. [5], we expect almost the same DM phenomenology as described in [7], with m_{DM} in the range $\text{few GeV} < m_{DM} < 100 \text{ GeV}$.

¹ For a similar change of basis in Q_6 see [16].

² We thank Luis Lavoura to point out this possibility.

IV. CHARGED LEPTONS AND NEUTRINOS

In the neutrino sector, the Dirac and Majorana mass matrices derived from eq.(4) read:

$$m_D = v_\eta \begin{pmatrix} y_1^\nu & y_1^\nu \\ y_2^\nu & -y_2^\nu \\ y_3^\nu & y_3^\nu \end{pmatrix} \quad M_R = M_1 \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad (20)$$

and the resulting light neutrino mass matrix, from type-I seesaw mechanism $m_\nu = -m_D M_R^{-1} m_D^T$, can be parametrized as:

$$m_\nu = \begin{pmatrix} 2A^2 & 0 & 2AC \\ 0 & -2B^2 & 0 \\ 2AC & 0 & 2C^2 \end{pmatrix}, \quad (21)$$

where $A^2 = (y_1^\nu v_\eta)^2/M_1$, $B^2 = (y_2^\nu v_\eta)^2/M_1$ and $C^2 = (y_3^\nu v_\eta)^2/M_1$. This matrix is diagonalized by:

$$U_\nu^T \cdot m_\nu \cdot U_\nu = D_\nu; \quad (22)$$

to find U_ν , we first compute the eigenvalues $|m_{\nu_i}|$ and eigenvectors of m_ν , related in the following way:

$$0 \rightarrow \begin{pmatrix} -\frac{|A|C^*}{A^* \sqrt{|A|^2+|C|^2}} \\ 0 \\ \frac{|A|}{\sqrt{|A|^2+|C|^2}} \end{pmatrix} \quad 2|B|^2 \rightarrow \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \quad 2(|A|^2 + |C|^2) \rightarrow \begin{pmatrix} \frac{AC^*}{C \sqrt{|A|^2+|C|^2}} \\ 0 \\ \frac{|C|}{\sqrt{|A|^2+|C|^2}} \end{pmatrix}. \quad (23)$$

The zero eigenvalue can be assigned to m_{ν_1} or m_{ν_3} . In the latter case, for any ordering of the remaining mass eigenstates, the solar angle cannot be reproduced. On the other hand, for $m_{\nu_1} = 0$, we get an appropriate U_ν if we associate $|m_{\nu_2}| = 2(|A|^2 + |C|^2)$ and $|m_{\nu_3}| = 2|B|^2$, with the condition $|B|^2 > (|A|^2 + |C|^2)$ to fit the atmospheric mass difference. The resulting U_ν is then given by:

$$U_\nu = \begin{pmatrix} -\frac{|A|C^*}{A^* \sqrt{|A|^2+|C|^2}} & \frac{AC^*}{C \sqrt{|A|^2+|C|^2}} & 0 \\ 0 & 0 & 1 \\ \frac{|A|}{\sqrt{|A|^2+|C|^2}} & \frac{|C|}{\sqrt{|A|^2+|C|^2}} & 0 \end{pmatrix} = \begin{pmatrix} c_\odot & s_\odot & 0 \\ 0 & 0 & 1 \\ -s_\odot & c_\odot & 0 \end{pmatrix} \quad (24)$$

with

$$\tan \theta_\odot = \left| \frac{A}{C} \right|. \quad (25)$$

Now we consider the charged sector. Since we work in the left-right basis, the mass matrices $M_\ell M_\ell^\dagger$ and $M_\ell^\dagger M_\ell$ are diagonalized by the unitary matrices U_ℓ and V_ℓ , respectively:

$$U_\ell^\dagger \cdot M_\ell M_\ell^\dagger \cdot U_\ell = D_\ell^2 \quad (26)$$

$$V_\ell^\dagger \cdot M_\ell^\dagger M_\ell \cdot V_\ell = D_\ell^2, \quad (27)$$

where D_ℓ is the diagonal charged lepton mass matrix.

After electroweak symmetry breaking the mass matrix of the charged leptons has the form:

$$M_l = \begin{pmatrix} y_1^l v' \varepsilon & 0 & 0 \\ 0 & y_2^l v \varepsilon & y_4^l v' \varepsilon \\ 0 & y_5^l v' & y_3^l v \end{pmatrix} \quad (28)$$

where we have defined $\langle \phi \rangle / \Lambda = \varepsilon$. The charged lepton masses are given by:

$$\begin{aligned} m_e &\approx y_1^l v' \varepsilon \\ m_\mu &\approx \varepsilon \frac{v^2 y_2^l y_3^l - v'^2 y_4^l y_5^l}{\sqrt{(v y_3^l)^2 + (v' y_5^l)^2}} \\ m_\tau &\approx \sqrt{(v y_3^l)^2 + (v' y_5^l)^2}. \end{aligned} \quad (29)$$

We can easily see that the muon mass is suppressed with respect to m_τ by a factor of ε and enhanced with respect to m_e by roughly a factor of v'/v , which is smaller than 1 (see below). If we concentrate only on the $\mu - \tau$ submatrix, we can rewrite it as:

$$M_{\mu\tau} = \begin{pmatrix} a & b \\ c & d \end{pmatrix}, \quad (30)$$

where a, b, c and d are products of Yukawa couplings and vevs. We can define two unitary rotations V_L and V_R

$$V_L = \begin{pmatrix} \cos \theta_\ell & \sin \theta_\ell \\ -\sin \theta_\ell & \cos \theta_\ell \end{pmatrix}, \quad V_R = \begin{pmatrix} \cos \theta_R & \sin \theta_R \\ -\sin \theta_R & \cos \theta_R \end{pmatrix} \quad (31)$$

such that:

$$V_L^\dagger M_{\mu\tau} V_R = \begin{pmatrix} m_\mu^2 & 0 \\ 0 & m_\tau^2 \end{pmatrix}. \quad (32)$$

Given the neutrino mixing matrix in eq.(24), it is evident that the atmospheric angle originates from V_L while the angle θ_R is a free parameter. The global 3×3 charged lepton mass matrix is then diagonalized by the following rotations:

$$U_\ell = \begin{pmatrix} 1 & 0 \\ 0 & V_L \end{pmatrix}, \quad V_\ell = \begin{pmatrix} 1 & 0 \\ 0 & V_R \end{pmatrix}. \quad (33)$$

To get an estimate of the magnitude of the Yukawa parameters in (28) we proceed as follows. The vevs v and v' are fixed from the minimization condition of the scalar potential, then the four Yukawa couplings $y_{2,3,4,5}^l$ (assumed to be real) are determined from the atmospheric angle, the μ and τ masses and the angle θ_R . Let us assume $\theta_\ell = \pi/4$ (in good agreement with the experimental data); then, the conditions for having a $\mu\tau$ invariant submatrix (30) can be deduced using:

$$M_{\mu\tau} M_{\mu\tau}^\dagger = V_L \begin{pmatrix} m_\mu^2 & 0 \\ 0 & m_\tau^2 \end{pmatrix} V_L^\dagger. \quad (34)$$

We get:

$$\begin{aligned} a^2 + b^2 &= c^2 + d^2 \\ c^2 + d^2 &= \frac{m_\tau^2 + m_\mu^2}{2} \\ db + ca &= \frac{m_\tau^2 - m_\mu^2}{2}. \end{aligned} \quad (35)$$

In this way we can write three out of four parameters (for instance, a, b and d) in terms of the charged lepton masses, the atmospheric angle (supposed to be maximal here) and c . The latter is related to the θ_R angle by:

$$\tan 2\theta_R = -\frac{2(ab + cd)}{a^2 - b^2 + c^2 - d^2}. \quad (36)$$

It is easy to show that the system of equations in (35) has real solutions only of the form $a \sim c$ and $b \sim d$ from which we deduce ³:

$$y_2^l v \varepsilon \approx y_5^l v', \quad y_3^l v \approx y_4^l v' \varepsilon. \quad (37)$$

For $y_2^l \sim y_5^l$ the first relation implies $v'/v \sim \varepsilon$ whereas the second one requires a moderate fine-tuning of order $y_3^l/y_4^l \sim \varepsilon^2$.

Finally, the lepton mixing matrix is given by:

$$U_{lep} = U_\ell^\dagger \cdot U_\nu = \begin{pmatrix} c_\odot & s_\odot & 0 \\ \sin \theta_\ell s_\odot & -c_\odot \sin \theta_\ell & \cos \theta_\ell \\ -\cos \theta_\ell s_\odot & s_\odot \cos \theta_\ell & \sin \theta_\ell \end{pmatrix} \quad (38)$$

where, considering the relations in eq.(37), gives:

$$\tan \theta_{23} \sim \frac{y_5^l}{y_3^l} \varepsilon. \quad (39)$$

To reproduce the correct maximal mixing in the atmospheric sector we need $y_3^l/y_5^l \sim \varepsilon$. We clearly see that our model predicts a vanishing θ_{13} . Since also $m_1^\nu = 0$, the effective mass entering the neutrinoless double beta decay assumes a particularly simple expression:

$$|m_{\beta\beta}| = |m_{\nu_2}| s_\odot^2 = \sqrt{\Delta m_\odot^2} s_\odot^2, \quad (40)$$

where $\Delta m_\odot^2 = |m_{\nu_2}|^2 - |m_{\nu_1}|^2$, with numerical values in the interval:

$$0.00054 \text{ eV} \leq |m_{\beta\beta}| \leq 0.0012 \text{ eV}. \quad (41)$$

V. ESTIMATE OF LEPTON FLAVOR VIOLATING PROCESSES

Since we have more than one $SU(2)$ Higgs doublet coupled to charged leptons, our model allows for lepton flavor violating processes (LFV) at tree level such as $\tau \rightarrow 3\mu$ and $\tau \rightarrow \mu ee$ (see [17] for other examples of renormalizable models based on dihedral groups). Here we are not interested in a full study of the LFV but just to show that the model prediction for them can be easily maintained below their upper bounds. In the interaction basis, the Yukawa matrices Y and Y' can be deduced from:

$$L \cdot Y \cdot l^c H + L \cdot Y' \cdot l^c H' = L \begin{pmatrix} 0 & 0 & 0 \\ 0 & y_2^l & 0 \\ 0 & 0 & y_3^l \end{pmatrix} l^c H + L \begin{pmatrix} y_1^l & 0 & 0 \\ 0 & 0 & y_4^l \\ 0 & y_5^l & 0 \end{pmatrix} l^c H', \quad (42)$$

where we have reabsorbed ε into y_1^l , y_2^l and y_4^l . Once we rotate the fields L and l^c to the mass basis, the new Yukawa matrices \tilde{Y} and \tilde{Y}' read:

$$\tilde{Y} = U_\ell^\dagger \cdot Y \cdot V_\ell, \quad \tilde{Y}' = U_\ell^\dagger \cdot Y' \cdot V_\ell. \quad (43)$$

They are not diagonal and contain non vanishing $\mu - \tau$ entries, as it can easily deduced using eq.(33). The Higgs fields should also be expressed in the mass basis but we do not take this additional rotation into account since it would only introduce additional mixing angles as suppression factors in the branching fraction computations (we are then working in the case where the LFV processes are the largest allowed in our model).

³ The relative hierarchy among the two groups of parameters depend on the choice of θ_R .

As explained above, the Yukawa couplings y_i are fixed from the value of fermion masses m_e , m_μ and m_τ , the vevs v and v'^4 (determined by the potential parameters) and the arbitrary angle θ_R of the V_ℓ unitary matrix. To get realistic estimates, we performed a numerical simulation with the constraints defined below eq.(12). It turns out that we can always find solutions with $v > v'$, which implies $\tilde{Y}' > \tilde{Y}$, see eqs.(37) and (42). In this case, the decay width for the $\tau \rightarrow 3\mu$ process⁵ is approximated by:

$$\Gamma(\tau \rightarrow 3\mu) \approx \frac{m_\tau^5 \left(\tilde{Y}'_{\mu\mu} \tilde{Y}'_{\tau\mu} \right)^2}{6 \times 2^9 \pi^3 m_{H'}^4}. \quad (44)$$

Numerical examples of the branching ratio $Br(\tau \rightarrow 3\mu)$ are given in Tab.II, where we choose the vev v , the ratio v/v' and $m_{H'}$ as independent variables. We fixed the value of θ_R to $\sin \theta_R = 0.9277$ for which the product of the Yukawas $\tilde{Y}'_{\mu\mu} \tilde{Y}'_{\tau\mu}$ is maximal and the branching ratios are the largest possible. As we can see, there is a region of the parameter

v (GeV)	v/v'	$m_{H'}$ (GeV)	m_H (GeV)	m_{DM} (GeV)	$Br(\tau \rightarrow 3\mu)$
224	3.6	140	115	89	8.1×10^{-9}
225	3.8	201	98	87	2.5×10^{-9}
225	3.8	175	132	60	3.1×10^{-9}
222	3.5	206	118	84	1.5×10^{-9}
173	1.5	266	223	75	5.4×10^{-11}

TABLE II: *Branching ratio for the process $\tau \rightarrow 3\mu$ as deduced from our model. The experimental bound is $Br(\tau \rightarrow 3\mu) < 3.2 \cdot 10^{-8}$ [18].*

space where the branching ratio for the tau decay is well below the experimental upper bound $Br(\tau \rightarrow 3\mu) < 3.2 \cdot 10^{-8}$ [18]. On the other hand, the first entry is very close to the upper limit, showing that a sector of the parameter space will be tested in the near future at the LHC.

VI. THE QUARK SECTOR

In this section we discuss the extension of our model to the quark sector. The assignment of the quark fields to the irreducible representation of D_4 is listed in Tab.III.

	Q_1	Q_2	Q_3	q_1^c	q_2^c	q_3^c
$SU(2)$	2	2	2	1	1	1
D_4	1 ₄	1 ₂	1 ₁	1 ₄	1 ₂	1 ₁
Z_2^f	+	-	+	+	-	+

TABLE III: *Quark assignments in our model.*

The Lagrangian for the down-type quarks reads as follows:

$$\begin{aligned} \mathcal{L}_{down} = & y_1^d Q_1 q_1^c H + y_2^d Q_1 q_2^c H' + y_3^d Q_2 q_1^c H' + y_4^d Q_2 q_2^c H + y_5^d Q_3 q_3^c H + \\ & + \frac{\phi}{\Lambda} (y_6^d Q_1 q_3^c H' + y_7^d Q_2 q_3^c H + y_8^d Q_3 q_1^c H' + y_9^d Q_3 q_2^c H). \end{aligned} \quad (45)$$

⁴ In the computation of the branching ratios, we set $\sqrt{2}v_\eta = v'$.

⁵ The process $\tau \rightarrow \mu ee$ is suppressed by \tilde{Y}'_{ee} .

For the up-type quarks the Lagrangian has the same structure with the obvious replacements $y_i^d \rightarrow y_i^u$ and $(H, H') \rightarrow (\tilde{H}, \tilde{H}')$, where $\tilde{H} = -i\tau_2 H^\dagger$. The mass matrices are then:

$$m_{u,d} = \begin{pmatrix} y_1^{u,d} v & y_2^{u,d} v' & y_6^{u,d} v' \varepsilon \\ y_3^{u,d} v' & y_4^{u,d} v & y_7^{u,d} v \varepsilon \\ y_8^{u,d} v' \varepsilon & y_9^{u,d} v \varepsilon & y_5^{u,d} v \end{pmatrix}. \quad (46)$$

With such a texture we can easily fit the quark masses and the CKM mixing angles. In particular, given that $v > v'$ (as discussed before eq.(44)), we can fix $y_{1,4,5}^{u,d}$ in such a way that $m_1^{u,d} \sim y_1^{u,d} v$, $m_2^{u,d} \sim y_4^{u,d} v$ and $m_3^{u,d} \sim y_5^{u,d} v$. Then the Cabibbo angle is given by:

$$\theta_C \sim \left(\frac{y_2^u}{y_4^u} - \frac{y_2^d}{y_4^d} \right) \frac{v'}{v}, \quad (47)$$

and it can be fit to its experimental value for a suitable choice of the vev ratio v'/v (with $y_2^{u,d}$ of the same order of $y_4^{u,d}$). Taking $y_{6,7}^{u,d}$ of the same order of $y_5^{u,d}$, we also have $V_{ub} \approx \varepsilon v'/v$ and $V_{cb} \approx \varepsilon$ (which fixes $\varepsilon \sim \mathcal{O}(0.04)$).

Since the Cabibbo mixing arises from both the up and down sectors, we can have $s-d$ and $c-u$ tree-level transitions mediated by Higgses. This implies that decays like $K^{+,0} \rightarrow \pi^{+,0} l \bar{l}$ (in the down sector) or $D^{+,0} \rightarrow \pi^{+,0} l \bar{l}$ and $D_s^+ \rightarrow K^+ l \bar{l}$ (in the up sector) can exceed their experimental bounds. Since the coupling \tilde{Y}'_{ee} is suppressed by the electron mass, the pairs $l \bar{l}$ can be $\mu^- \mu^+$ whereas the case $\mu^\pm \tau^\mp$ is kinematically excluded. The tree-level transitions $b-s$, $b-d$ are suppressed by ε and we only consider $B^+ \rightarrow K^+ \mu^+ \mu^-$ and $B^+ \rightarrow \pi^+ \mu^+ \mu^-$. In order to give an estimate of such processes, we work in the worst case of unity mixings and (adimensional, that is stripped of the meson masses) form factors. We then have:

$$\begin{aligned} \Gamma(K^{+,0} \rightarrow \pi^{+,0} \mu^+ \mu^-) &\approx \frac{1}{3072\pi^3} \left(\frac{m_K^5}{m_{H'}^4} \right) |\tilde{Y}'_{ds} \tilde{Y}'_{\mu\mu}|^2, \\ \Gamma(D^{+,0} \rightarrow \pi^{+,0} \mu^+ \mu^-) &\approx \frac{1}{3072\pi^3} \left(\frac{m_D^5}{m_{H'}^4} \right) |\tilde{Y}'_{cu} \tilde{Y}'_{\mu\mu}|^2, \\ \Gamma(D_s^+ \rightarrow K^+ \mu^+ \mu^-) &\approx \frac{1}{3072\pi^3} \left(\frac{m_{D_s}^5}{m_{H'}^4} \right) |\tilde{Y}'_{cu} \tilde{Y}'_{\mu\mu}|^2. \\ \Gamma(B^+ \rightarrow K^+ \mu^+ \mu^-) &\approx \frac{1}{3072\pi^3} \left(\frac{m_B^5}{m_{H'}^4} \right) |\tilde{Y}'_{bs} \tilde{Y}'_{\mu\mu}|^2. \\ \Gamma(B^+ \rightarrow \pi^+ \mu^+ \mu^-) &\approx \frac{1}{3072\pi^3} \left(\frac{m_B^5}{m_{H'}^4} \right) |\tilde{Y}'_{bd} \tilde{Y}'_{\mu\mu}|^2. \end{aligned} \quad (48)$$

We observe that for $v \sim 224$ GeV (the worst point in Tab.II), the couplings $y_{2,3,4,5}^l$ in eq.(42) should be of $\mathcal{O}(10^{-2})$ to reproduce the τ mass and then $\tilde{Y}_{\mu\mu} \sim 10^{-2}$. In the down sector we have $y_5^d \sim 2 \cdot 10^{-2}$, $y_4^d \sim 6 \cdot 10^{-4}$, $y_1^d \sim 3 \cdot 10^{-5}$ and $y_2^d \sim y_4^d$ and therefore $\tilde{Y}'_{ds} \sim 6 \cdot 10^{-4}$. A similar reasoning in the up sector gives $\tilde{Y}'_{cu} \sim 7 \cdot 10^{-3}$. Using the above values, we computed the branching ratios for the meson decay processes in eq.(48). They are summarized in Tab.IV.

We clearly see that all branching ratios are below their upper limits. We have also checked that the mass difference in the kaon system, driven by the $K^0 - \bar{K}^0$ oscillation, is around 10^{-14} GeV, to be compared with the experimental value $\sim 10^{-12}$ GeV. We stress that, even if we have taken a particular point in the parameter space to make our estimates, the exercise can be repeated for different input values with similar conclusions.

VII. CONCLUSIONS

In this paper we have discussed an extension of the standard model based on the non-abelian discrete group D_4 . We introduced three more $SU(2)$ Higgs doublets, a combination of them giving a good dark matter candidate, one standard model singlet and only two right-handed neutrinos, a remarkable feature if compared with the models in

<i>decay</i>	model prediction	experimental bounds [18]
$\text{Br}(K^+ \rightarrow \pi^+ \mu^+ \mu^-)$	$5.5 \cdot 10^{-9}$	$< 8.1 \cdot 10^{-8}$
$\text{Br}(K^0 \rightarrow \pi^0 \mu^+ \mu^-)$	$1.2 \cdot 10^{-9}$	$< 2.9 \cdot 10^{-9}$
$\text{Br}(D^+ \rightarrow \pi^+ \mu^+ \mu^-)$	$1.3 \cdot 10^{-8}$	$< 3.9 \cdot 10^{-6}$
$\text{Br}(D^0 \rightarrow \pi^0 \mu^+ \mu^-)$	$5.2 \cdot 10^{-9}$	$< 1.8 \cdot 10^{-4}$
$\text{Br}(D_s^+ \rightarrow K^+ \mu^+ \mu^-)$	$8.3 \cdot 10^{-9}$	$< 3.6 \cdot 10^{-5}$
$\text{Br}(B^+ \rightarrow K^+ \mu^+ \mu^-)$	$6.4 \cdot 10^{-8}$	$< 5.2 \cdot 10^{-7}$
$\text{Br}(B^+ \rightarrow \pi^+ \mu^+ \mu^-)$	$3.2 \cdot 10^{-10}$	$< 1.4 \cdot 10^{-6}$

TABLE IV: *Branching fraction estimates for some interesting processes in our model. We fixed $m_{H'} = 140$ GeV, $v = 224$ and $\tilde{Y}'_{\mu\mu} = 0.0145$.*

[5–7]. The stability of the DM candidate is not imposed ad-hoc but directly follows from the remnant Z_2 subgroup of the broken D_4 , as we explained in details. Within the same framework, we incorporated a description of the charged leptons and neutrinos, showing that the normal hierarchy (with $m_{\nu_1} = 0$) and a vanishing θ_{13} are natural predictions of our model. This also allows to get a range of values for the effective mass $m_{\beta\beta}$, which turns out to be in the interval $[0.5, 1.2] \cdot 10^{-3}$ eV. On the other hand, the solar and atmospheric angles are free parameters that can be easily fixed to their corresponding experimental values. We have carefully checked that, in a large portion of the parameter space, the model does not conflict with the upper bounds on some lepton flavor violating processes, like $\tau \rightarrow 3\mu$. Finally, we extended the D_4 symmetry to the quark sector, showing that the correct order of magnitude for the CKM angles can be easily reproduced assigning the quark fields to non trivial representation of D_4 . Three-level flavor changing neutral current processes, generated by non-vanishing off diagonal Yukawa couplings, can be maintained below their current experimental bounds.

VIII. ACKNOWLEDGMENTS

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